

# Non-Parametric Polygenic Risk Prediction

Shamil Sunyaev

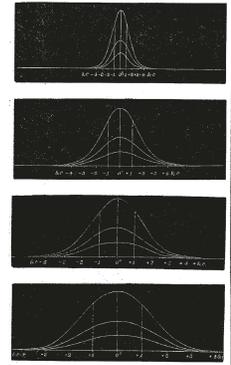


NATURE

[April 5, 1877]

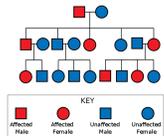
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**TYPICAL LAWS OF HEREDITY<sup>1</sup>**  
**WE** are far too apt to regard common events as matters of course, and to accept many things as obvious truths which are not obvious truths at all, but present problems of much interest. The problem to which I am about to direct attention is one of these.

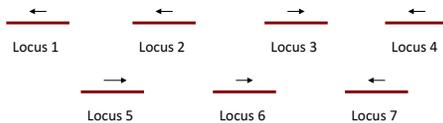
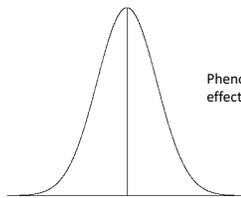


## Quantitative Trait Loci (QTLs)

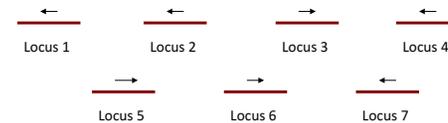
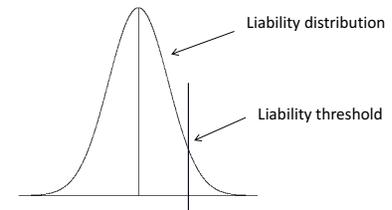
Inheritance at each locus is Mendelian. Loci are independent



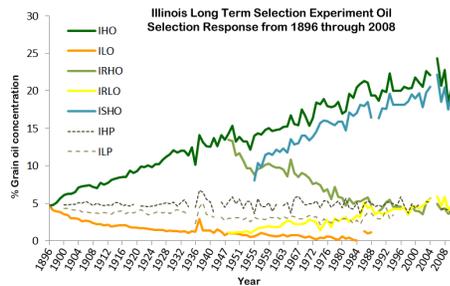
Phenotype is additive over locus effects -> normal distribution



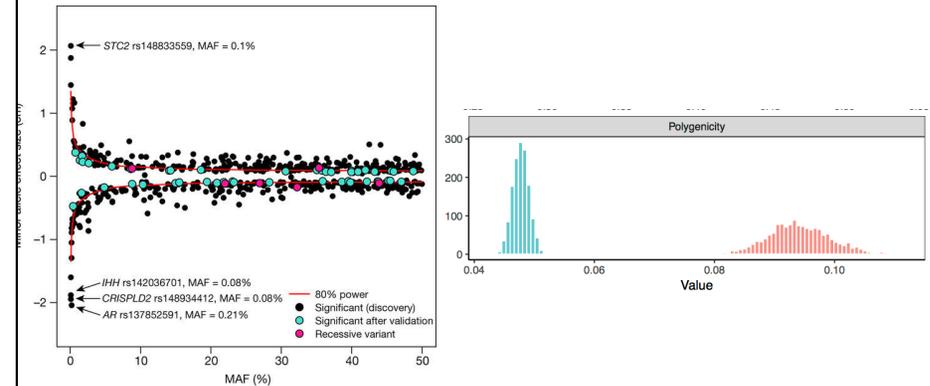
## Binary traits such as diseases



## Early evidence of high polygenicity of complex traits



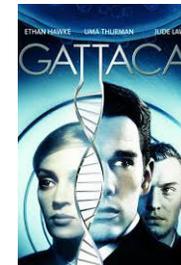
## Evidence in favor of the highly polygenic model



## Effect sizes of individual variants are very small

- Genotype at a single locus carries very little information about phenotype.
- It does not mean that one cannot predict phenotype from genotype.
- Accuracy ( $r^2$ ) of an ideal genetic predictor equals heritability.

## Genetic risk prediction



Genotype of an individual (Common SNPs) → Life-time risk of genetic disorders (Common complex genetic disorders)

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## Measuring risk of myocardial infarction

### Coronary Risk Prediction in Adults (The Framingham Heart Study)

PETER W.F. WILSON, MD, WILLIAM P. CASTELLI, MD,  
and WILLIAM B. KANNEL, MD

The Framingham Heart Study, an ongoing prospective study of adult men and women, has shown that certain risk factors can be used to predict the development of coronary artery disease. These factors include age, gender, total cholesterol level, high density lipoprotein cholesterol level, systolic blood pressure, cigarette smoking, glucose intolerance and cardiac enlargement (left ventricular hypertrophy on electrocardiogram or enlarged heart on chest x-ray). Calculators and computers can be easily programmed using a multivariate logistic

function that allows calculation of the conditional probability of cardiovascular events. These determinations, based on experience with 5,209 men and women participating in the Framingham study, estimate coronary artery disease risk over variable periods of follow-up. Modeled incidence rates range from <1% to >80% over an arbitrarily selected 6-year interval; however, they are typically <10%, and rarely exceed 45% in men and 25% in women.

(Am J Cardiol 1987;59:91G-94G)

## LDL levels and risk of disease

Annals of Internal Medicine

ARTICLE

### Nonoptimal Lipids Commonly Present in Young Adults and Coronary Calcium Later in Life: The CARDIA (Coronary Artery Risk Development in Young Adults) Study

Mark J. Pletcher, MD, MPH; Kirsten Bibbins-Domingo, PhD, MD; Kiang Liu, PhD; Steve Sidney, MD, MPH; Feng Lin, MS; Eric Vittinghoff, PhD; and Stephen B. Hulley, MD, MPH

~3500 subjects < 35 years old

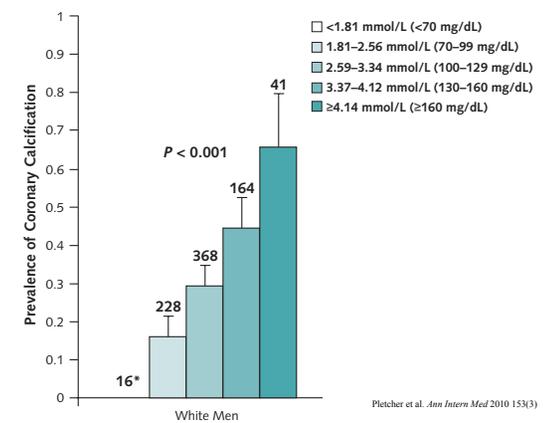


15-20 years

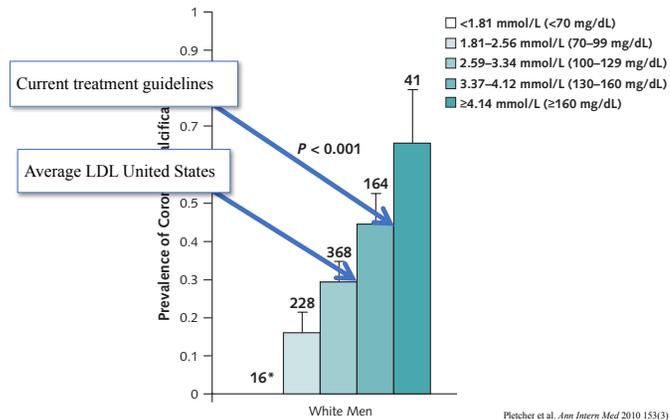


Peters et al. BMC Cardiovascular Disorders 2008  
8:38

## LDL levels and risk of disease



## LDL levels and risk of disease



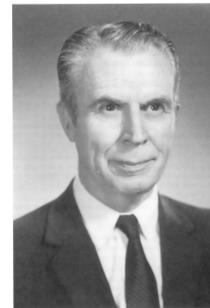
## Selecting populations for treatment



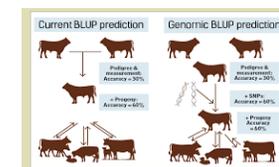
## Why estimate genetic risk?

- An estimate of the long-term risk at birth
- Genetic risk can be combined together with biomarkers and clinical features
- Genetics explains about 50% of risk. One cannot predict risk any better than that but 50% is a non-trivial proportion of risk

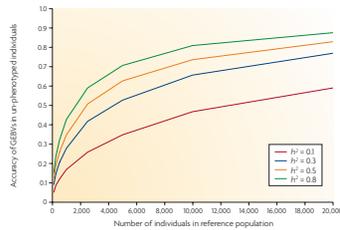
## BLUP – Best Linear Unbiased Predictor



- Infinitesimal model
- Genetic effects are random
- Predict the expected genetic effect



## Accuracy of polygenic prediction in cattle



Poor transferability between breeds!

## Applications in humans



### Prediction of individual genetic risk to disease from genome-wide association studies

Naomi R. Wray, Michael E. Goddard and Peter M. Visscher

*Genome Res.* 2007 17: 1320-1328, originally published online Sep 4, 2007;

Access the most recent version at doi:10.1101/gr.666407

### LETTERS

#### Common polygenic variation contributes to risk of schizophrenia and bipolar disorder

The International Schizophrenia Consortium\*

- LD-prune
- Exclude SNPs of very small effect

## Extensions of BLUP – multiple variance scales and binary phenotypes

MultiBLUP:	Speed and Balding. <i>Genome Research</i> 2014
Bayesian analysis:	MacLeod et al. <i>Genetics</i> 2014
BSLMM:	Zhou et al. <i>PLOS Genetics</i> 2013
GeRSI:	Golan and Rossett. <i>AJHG</i> 2014

## Methods that work with summary statistics

- Summary statistics are easily available
- Most methods require a separate small individual level dataset to tune parameters

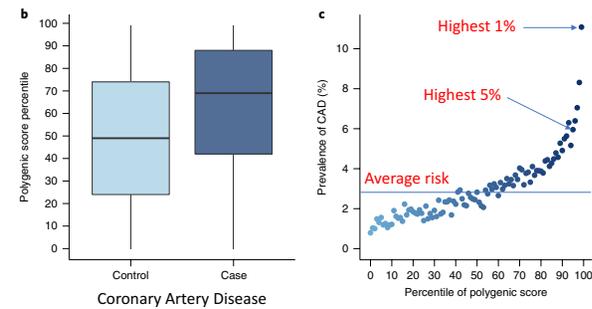
## LDPred – a Bayesian method using summary statistics

$$\beta_i \sim_{iid} \begin{cases} N\left(0, \frac{h_s^2}{Mp}\right) & \text{with probability } p \\ 0 & \text{with probability } (1-p), \end{cases}$$

Vilhjalmsson et al. 2015

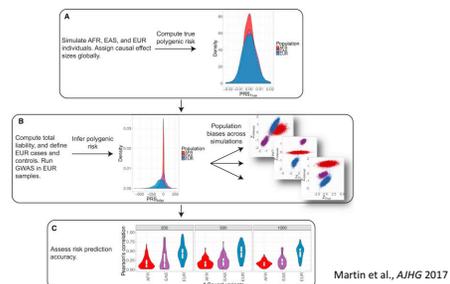
Also, check *BayesR*

Extreme tails in the distributions of genetic risk scores are highly predictive



Khera et al. 2018

With some caveats



Martin et al., AJHG 2017

Linear models for genetic risk prediction

$$y_i = \sum_j \beta_j x_{ij}$$

Genetic risk of individual  $i$       Effect size of SNP  $j$       Genotype of SNP  $j$  and individual  $i$

“Polygenic scores” can leverage summary statistics from a large GWAS study

$$\hat{y}_i = \sum_j \hat{\beta}_j x_{ij}$$

Predicted genetic risk

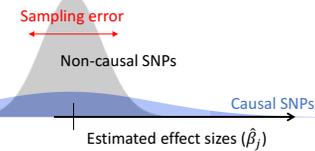
Estimated effect size

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Predicted genetic risk

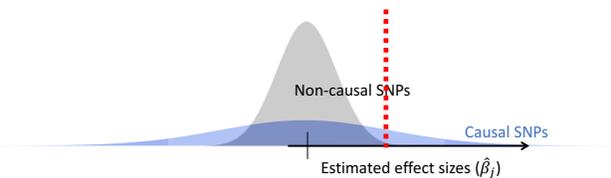
Estimated effect size



“Polygenic scores” can leverage summary statistics from a large GWAS study

P-value Thresholding

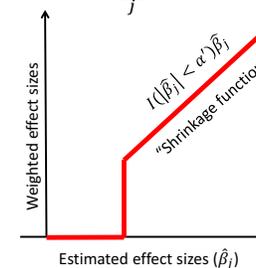
$$\hat{y}_i = \sum_j \hat{\beta}_j x_{ij}$$



P-value thresholding can be reformulated as “shrinking” estimated effect sizes

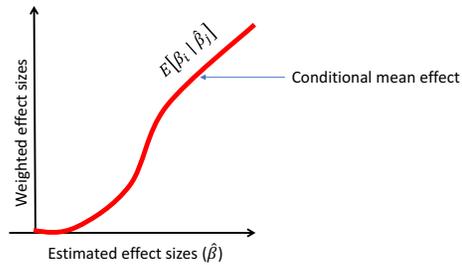
P-value Thresholding

$$\hat{y}_i = \sum_j I(|\hat{\beta}_j| < \alpha') \hat{\beta}_j x_{ij}$$



The optimal polygenic score can be constructed with “conditional mean effects”

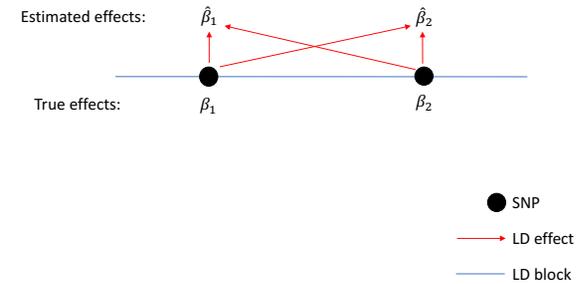
$$\hat{y}_i = \sum_j E[\beta_j | \hat{\beta}_j] x_{ij}$$



Goddard et al. 2009

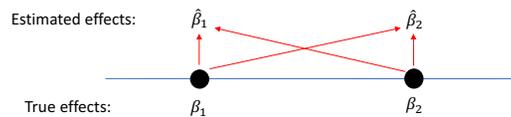
Accounting for LD in summary data is a major challenge

- Correlation between **apparent true genetic effects**



Accounting for LD in summary data is a major challenge

- Correlation between **apparent true genetic effects**



- Correlation between **sampling errors**



Our approach (“Non-Parametric Shrinkage” or NPS)

- No explicit specification of genetic architecture prior, thus “*non-parametric*”
- Learn conditional mean effects directly from training data
- Fully account for correlation in summary statistics

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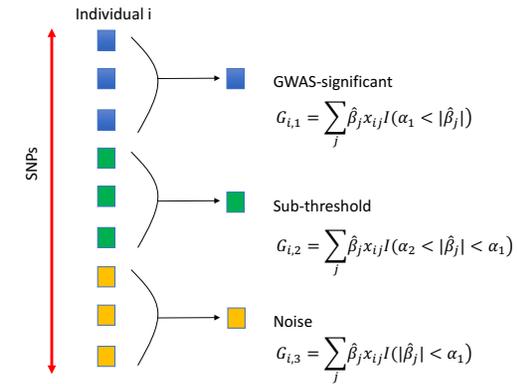
- Learn conditional mean effects directly from training data

1. How to estimate  $E[\beta_j | \hat{\beta}_j]$  without a Bayesian prior on  $\beta$

- Fully account for correlation in summary statistics

2. How to deal with LD

## Partitioned risk scores

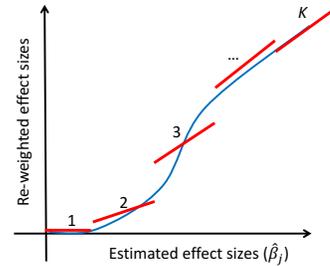
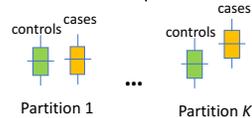


## Piecewise linear interpolation on shrinkage curve

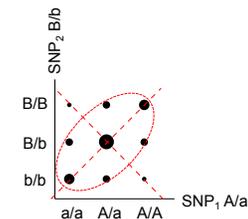
Estimates of genetic effects in GWAS data ( $\hat{\beta}_j$ )

Partition SNPs into  $K$  subgroups:  
 $S_k = \{j : b_{k-1} < |\hat{\beta}_j| < b_k\}$

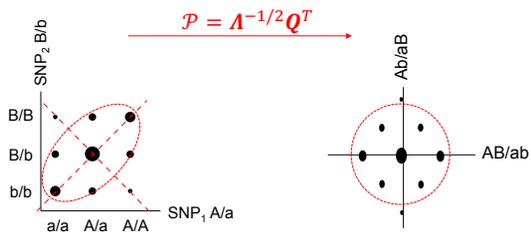
Partitioned risk scores:  $G_{ik} = \sum_{j \in S_k} \hat{\beta}_j x_{ij}$



## How to deal with LD?



### Decorrelating linear projection $\mathcal{P}$

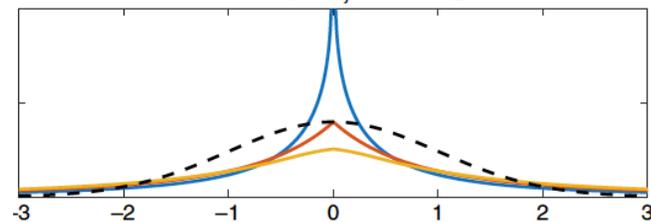


$\Sigma$  is a local LD matrix and  $\Sigma = Q \Lambda Q^T$  by eigenvalue decomposition  
 $\Sigma^{-1} = Q \Lambda^{-1} Q^T = (Q \Lambda^{-1/2})(\Lambda^{-1/2} Q^T)$

### Other shrinkage methods: PRS-CS

$$\beta_j \sim N\left(0, \frac{\sigma^2}{N} \phi \psi_j\right), \quad \psi_j \sim g,$$

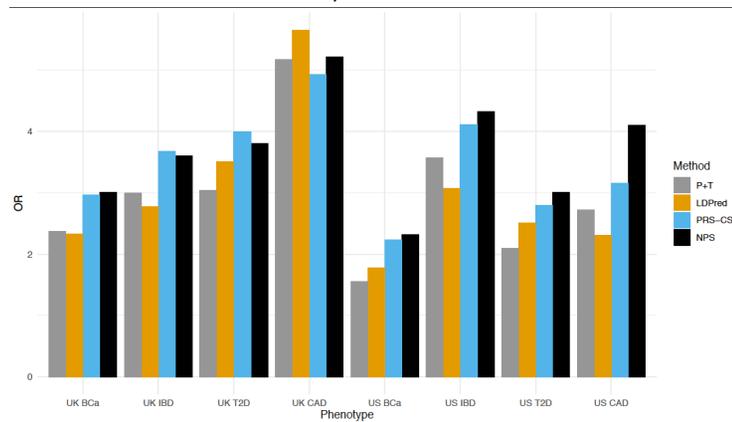
Prior density of  $\beta_j$ ; central region



### Other shrinkage methods: PRS-CS

*Lassosum* – extension of *LASSO*

### Accuracy of the 5% tail



## Summary on the method

- NPS accounts for the correlation of sampling errors in GWAS summary statistics.
- NPS provides an extensible framework to estimate the shrinkage curve from training data.
- NPS is best-suited to take advantage of the high density of markers and imputation accuracy in latest GWAS datasets.

The preprint is available in BioRxiv:  
Chun et al. "*Non-parametric polygenic risk prediction using partitioned GWAS summary statistics.*"  
Software is available at: <https://github.com/sgchun/nps>

## Is an extreme presentation with a family history Mendelian?

- It is often assumed that an extreme phenotypic presentation is due to a large effect Mendelian mutation.
- Apparently Mendelian family history is assumed to support a highly penetrant Mendelian mutation.
- Could these cases be polygenic (or, at least, not monogenic)?